

PROCEEDINGS

2nd COLLOQUIUM ON
THE DIDACTICS OF MATHEMATICS

DEPARTMENT OF EDUCATION
UNIVERSITY OF CRETE

FRIDAY - SATURDAY
21-22 APRIL 2000

UNIVERSITY CAMPUS OF RETHYMNON, CRETE

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PREFACE

This volume contains the papers which were presented during the *2nd Colloquium on the Didactics of Mathematics*. The *Colloquium* was organized by the Department of Education of the University of Crete at Rethymnon Crete, from 21 till 22 April 2000.

The aim of this *Colloquium* was to present the current international trends on important issues of the Didactics of Mathematics (DM) which are related mainly to the primary and secondary level of education. It was also intended to provide an opportunity for fruitful communication between the Greek researchers of the DM and their distinguished colleagues from abroad. Finally, it aimed at giving an opportunity to the educational community to be informed about those issues that are of great importance nowadays. This is one of the basic reasons why, alike the *1st Colloquium*, the proceedings contain both the original and the greek translations of all presentations of the foreign invited speakers. What is more, there is an abstract in either English or French for every paper written in Greek, so as the non-greek speaker to be able to have an overall view of the content of all the papers.

We would like to thank the members of the Scientific Committee who were willing to review the submitted papers thoroughly. We also thank the Rectorate and the Dean of the Faculty of the Sciences of Education of the University of Crete for their moral and material support.

Every paper falls within one or more of the five sections of this volume. For practical reasons, each one is cited in the section with the content of which it is related more: (a) Learning and failure in Mathematics, (b) Aspects of the Didactics of Geometry, (c) Mathematics Teaching and Informatics, (d) Epistemological and Methodological issues on Mathematics and its Teaching, (e) Issues on the Philosophy and the History of Mathematics. The papers suggest clearly that the DM issues ask for an interdisciplinary approach, since they are related to different areas, such as Mathematics proper, its History and Epistemology, its relationship with the other exact sciences - Physics in particular - Cognitive Psychology, Applied Statistics, Informatics etc. Consequently, to understand and improve teaching and learning, both the researcher and the Mathematics teacher of all levels should go deep into more than one of the aforementioned areas. Inferently, it arises that, to become more effective, the teachers of all levels, from the primary to the tertiary, should aim at a broader education that goes beyond Mathematics. Apart from the above, the researcher of the DM, must have a solid knowledge in a broader area of Mathematics, its Epistemology and its relationship with other disciplines and Psychology, in order for his inferences to be more valid.

Therefore, it is highlighted the necessity of a closer relation among the Departments of Education, the Departments of Mathematics and those primary and secondary school teachers, who try to teach Mathematics effectively, often under hard conditions. This becomes more important nowadays, because there is an international tendency not only towards Mathematics but education in general, for a superficial approach to a variety of subjects which does not facilitate their complete and deep understanding and consequently their learning.

We all desire and wish, this volume to promote a closer cooperation among all those interested in the improvement of Mathematics Education.

Rethymnon, November 2000

M. Kourkoulos
G. Troulis
C. Tzanakis

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