

5th INTERNATIONAL COLLOQUIUM ON THE DIDACTICS OF MATHEMATICS

DEPARTMENT OF EDUCATION

UNIVERSITY OF CRETE

THURSDAY 17 - SATURDAY 19 APRIL 2008

UNIVERSITY CAMPUS

RETHYMNON, CRETE

AUDITORIA Δ7, Δ6, Γ2

PROGRAMME & ABSTRACTS

THEMES

- 1. Aspects of the didactics of Arithmetic and Algebra*
- 2. The teaching and learning of Geometry*
- 3. Historical and epistemological issues in Mathematics Education – Revisions in Mathematics Education*
- 4. The teaching and learning of Statistics and Probability*
- 5. Alternative methods of teaching Mathematics*

INVITED SPEAKERS

Abraham Arcavi

Michèle Artigue

Bill Barton

Alain Kuzniak

Man Keung Siu

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TITLES OF PLENARY LECTURES

Abraham Arcavi, *Towards thinking and sense-making classroom practices: The case of Japanese mathematics lessons*

Michèle Artigue, *Digital technology in mathematics education: an instrumental approach*

Bill Barton, *Revisiting Felix Klein's "Elementary Mathematics From An Advanced Standpoint"*

Alain Kuzniak, *Diversity of geometrical paradigms and trouble in the teaching and learning of geometry*

Man Keung Siu, *The world of geometry in the classroom : virtual or real?*

TITLES OF PRESENTATIONS IN PARALLEL SESSIONS

(underlined is the name of the co-author(s) who will present the paper orally and whose name(s) appear(s) underlined on the time schedule of the programme)

S. Anastasiadou *A survey of research on the role of representations in the learning of statistical and probabilistic concepts at all levels of education (in Greek)*

M. Andresen, S. Froelund *Philosophical reflections made explicit as a tool for mathematical reasoning*

M. Bako, L. Aszalos *Play and learn with GCompris*

Th. Barrier *Quantification et variation en mathématiques: Perspectives didactiques issue de la lecture d'un texte de Bolzano*

S. Baştürk *Concours d'entrée à l'université en Turquie et ses effets négatifs sur les apprentissages des élèves de seconde*

V. Battie *le théorème fondamental de l'arithmétique: une approche historique et didactique*

A. Constantinides, N. Georgiou, M. Miltiadou *The use of manipulatives in the teaching of mathematics as expressed in teachers' beliefs (in Greek)*

C. Bonotto *About the role of real-world knowledge and experiences in mathematical modelling*

L. Bowie *Definitions in geometry in a South African textbook*

M. S. Britt, K. C. Irwin *A pathway for algebraic thinking*

E. Castagnola, R. Tortora *Can any definition of a triangle be considered the best one?*

D. Chassapis *The influence of a cultural tool on approaching a problem from the history: solving a geometry problem on graph paper*

A. Cusi, N. A. Malara *Future teachers facing proof problems: Games of interpretation, anticipating thought and coordination between verbal and algebraic register*

J-J. Dahan *Les paramètres didactiques cruciaux pour comprendre l'intégration de l'expérimental dans la pratique et l'enseignement de la géométrie: Exemplification grâce à Cabri 2 Plus et Cabri 3d*

K. Dalakoura *The mathematical education of women in Greece (19th-early 20th century) (in Greek)*

J. M. Delire *Les découvertes Keralaises sur les séries trigonométriques et leurs justifications dans le tantrasangraha (1500 d.n.e.) et ses commentaires*

G. Dimakos, E. Nikoloudakis *Teaching geometry in secondary school using van Hiele's theory and ICT in the context of cooperative learning (in Greek)*

E. Demetriadou, C. Tzanakis *Reinstatement of vectors in lower secondary education: some didactical suggestions and comments (in Greek)*

W. Dörfler *Learning mathematics: a naturalistic view*

Ch. Fili *Cyparissos Stephanos and the Erlanger Programme (in Greek)*

B. Georgiadou-Kambouridou **I. Antoniou**: *Students with low performance and the multiplication operation (in Greek)*

S. Götz *Fundamental Ideas and Basic Beliefs in Stochastics*

N. Hadas, **R. Hershkowitz**, **G. Ron** *Instructional design and research design principles in probability*

G. Halepaki: *Characteristics of 11-year-old students in connection with the estimation and control criteria of the outcome of arithmetic operations (in Greek)*

M. Hejný *Scheme and its generic models*

D. Iannece, **P. Romano** *What does it mean for a teacher to have a “scientific approach”? A reflection*

U. Th. Jankvist *Evaluating a teaching module on the early history of error correcting codes*

I. Kanellos *Findings from a diagnostic test given to students in the 1st year of the Greek Lyceum in 2006-07 (in Greek)*

H. Kasoumi, **C. Zaharos**, **M. Poulou** *Reception of the reformed mathematics curriculum by the nursery teachers of public education in Greece (in Greek)*

B. M. Kinach *Modelling tasks for developing geometry teachers’ spatial, epistemological, and pedagogical knowledge*

B. Koichu *On composing multiple-choice tasks, thought experimentation and algebra teachers’ knowledge base*

J. Kos *TRICK*

N. Kountourakis, **G. Polizois** *Teaching linear dynamic systems in elementary school, using simulations: Sensitive dependency on the initial conditions (in Greek)*

M. Kourkoulos *Didactical investigation of a simple physical model of moving particles for improving the understanding of variance in statistics (in Greek)*

Ch. Lemonidis, **F. Karannikis** *The notion of proportion in vocational activity and its relation to school experience: a case study with Cypriot bakers from Lemessos province (in Greek)*

M. Marmara, **C. Hatzikyriakou** *The use of algebra tiles in the teaching and learning of algebraic concepts (in Greek)*

Y. Martin *Dynamic implementation of Moulton’s plane (non-Desarguesian Geometry)*

M. Meletiou-Mavrotheris, **E. Mavrotheris**, **E. Paparistodemou** *EarlyStatistics: an online professional development program for European teachers of statistics*

H. Michail, **P. Sofokleous**, **Ch. Lemonidis** *The multicultural dimension of multiplication: attitudes and perceptions of Cypriot educators about its use in teaching mathematics (in Greek)*

Y. Michalis *The didactics of statistics in compulsory education (in Greek)*

E. Mokos, **S. Kafousi**, **F. Kalavasis** *The research on metacognition and the learning of mathematics: facts and perspectives (in Greek)*

K. Nikolantonakis *One, zero and infinity: three cases of historical constructions in the context of teachers’ training (in Greek)*

E. Nikoloudakis, **S. Ferentinos**, **E. Choustoulakis** *Overcoming obstacles in geometry by deploying representations of ICT (in Greek)*

E. Paparistodemou, **M. Meletiou-Mavrotheris** *Developing statistical reasoning in the early grades: a new role of technology*

S. Patsiomitou, **E. Koleza** *The development of students’ geometrical thinking through linking visual active representations*

G. Perikleidakis: *Exploring the performance of 9 - year - old elementary school students in doing arithmetical operations and problem solving during the initial implementation of the new textbooks and*

reformed curricula (in Greek)

A. Poulos *Classroom activities based on the concept of a Pythagorean triplet (in Greek)*

N. Prusak, R. Hershkowitz, N. Hadas, *From visual to logical argumentation within intentional designed activity*

M. Rugelj *How to “domesticate” logarithms in school? Giving sense to the logarithmic function*

N. Sgreccia, M. Massa *Relations between “saying” and “practice” of mathematics teachers when they teach geometry to students aged 12-15*

J. Slezáková *Stepping and ladder methods in the process of building up an additive triad schema and other mathematical schemas*

I. Spantidakis, H. Vasilaki *Writing as a problem solving process and anxiety levels of students with writing difficulties (in Greek)*

N. Stehlíková *Using video cases to promote good practice in the teaching of mathematics*

P. Svrckova *Teaching mathematics through pupils' projects*

Y. Thomaidis (Workshop) *Snapshots and pictures from integrating the history of mathematics in the new textbooks of secondary school in Greece (in Greek)*

A. Tsigoni *Trade world's contribution to the spreading of mathematics education in Ioannina throughout the Ottoman period (in Greek)*

H. Vasilaki, E. Giahnakis *Investigation of the relationship between working memory, anxiety and performance during the completion of an arithmetic mnemonic task (in Greek)*

C. Voica, F. M. Singer *From real life to probability laws: Integrating problem solving in teaching statistics and probabilities*

İ. Yavuz *Effets d'un changement de programme sur la prise en compte de nouveaux objets dans l'enseignement des fonctions*

J. Zhouf *Problem posing through correspondence seminars*

PLENARY LECTURES OF INVITED SPEAKERS: ABSTRACTS

Theme 5

Towards thinking and sense-making classroom practices: the case of Japanese mathematics lessons

Abraham Arcavi

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Japanese mathematics education has been the focus of much attention and interest, especially during the last decade. This presentation is aimed as a further contribution to the understanding of Japanese mathematics teaching by the worldwide mathematics education community. I will describe (through the eyes of an external observer) the most salient mathematical and pedagogical characteristics of the many lessons I have observed, with an emphasis on mathematical thinking and sense-making. I will discuss the generalizability of the principles underlying this tradition and some of the dilemmas thereof.

Theme 5

Digital technologies and mathematics education: an instrumental approach

Michèle Artigue

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For more than three decades mathematics education has tried to benefit from the use of digital technologies. After briefly evoking my personal trajectory as a university teacher and didactician in that area, I will focus on what is today known as the instrumental approach to the integration of digital technologies to which development I have contributed since 1995. First, I will present the beginning of this approach in the context of CAS, elucidating the needs, which I tried to respond to, and showing how it changed my personal view on technological issues. Then I will focus on more recent developments of this approach, due to its increasing use with other technologies such as dynamic geometry software, spreadsheets and on-line tutorial resources, and also due to a move in research interests from the student to the teacher.

Theme 3

Revisiting Felix Klein's *Elementary mathematics from an advanced standpoint*

Bill Barton

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A century ago, in 1908, Felix Klein's lectures on mathematics for secondary teachers were first published (in German). This comprehensive view of the field challenged both teachers and mathematicians to consider, from a perspective sensitive to both mathematical rigor and pedagogical practice, the relationship between mathematics as a school subject, and mathematics as a scientific discipline. The intervening 100 years have witnessed many changes in mathematics; the crises in Foundations, the advent of computing, emergence of new fields, and resolutions of some major mathematical challenges. These have provoked major changes and challenges for school mathematics. While Klein's writing remains a valuable source insight, it seems timely to revisit this terrain by linking the topics and approaches of senior secondary or undergraduate mathematics with the field of mathematics. This is an important challenge for both mathematicians and mathematics educators. This presentation will describe a writing project, possibly a joint project between IMU and ICMI, to revisit this work in a contemporary context.

Theme 2

Diversity of geometrical paradigms and trouble in the teaching and learning of geometry

Alain Kuzniak

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One of the principal aims of the teaching of geometry is certainly that students achieve to build their own proper and effective geometrical working space. Then they can understand and solve geometry problems by using this working space. But, the problem's interpretation depends on different geometrical paradigms related to various institutions – schools and also countries – where geometry is taught. From the diversity of paradigms it results a great diversity of working spaces: that explains a big number of didactic misunderstandings and learning difficulties. During the lecture, we will clarify the notions of geometrical paradigm and geometrical working space and show the possible interest of using and developing these tools for studies in geometry didactics.

Theme 2

The world of geometry in the classroom: Virtual or real?

Man-Keung Siu

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Despite the occurrence of the word “virtual” in the title, this is not a lecture on DGE (dynamic geometry environment), albeit I may have occasion to touch on that at some point. This lecture discusses the significance and the meaning of studying geometry in school, the content and evolution of the subject, in particular, attempts to design in the classroom a “world of geometry” that bridges the virtual (abstract) world and the real (concrete) world. It is expected that in this way, students will feel more at home instead of being alienated from the subject.

ABSTRACTS OF PRESENTATIONS IN PARALLEL SESSIONS

Theme 4

A survey of research on the role of representations in the learning of statistical and probabilistic concepts at all levels of education

Sofia Anastasiadou

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The understanding of a statistical or probabilistic concept presupposes the ability to recognize the concept when it is presented with a series of qualitatively different representation systems, the ability to flexibly handle the concept in the specific representation systems and finally, the ability to translate the concept from one system to another. In statistical education, the interest focuses both on the various types of representation and on the translations between them.

In this study, we revisited the role of representations in an effort to further understand the nature and structure of representations in developing statistical and probabilistic concepts. We investigated the developmental nature of the ability to use multiple representations and translate from one representation to another in relation to primary, secondary and tertiary Greek students. Although that the examine researches vary doth in context and research methodology they showed students difficulty in handling two or more representations in statistical or probabilistic tasks. This incompetence is the main feature of the phenomenon of compartmentalization in representations, which was detected in primary, secondary and tertiary students.

Theme 5 &3

Philosophical reflections made explicit as a tool for mathematical reasoning

Mette Andresen, Sune Froelund

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A new construct, ‘multidisciplinarity’, is prescribed in the curricula of Danish Upper Secondary Schools by governmental regulations since 2006. Multidisciplinarity offers a good chance to introduce philosophical tools or methods in mathematics with the aim to improve the students’ learning of both subjects, and to study the students’ reactions and signs of progressive mathematizing. Based on realistic mathematics education (RME), which is rooted in Hans Freudenthal’s idea of mathematics as a human activity, we decided to centre our work on the concept of reflection and to build a model for making students’ reflections in the mathematics class explicit to themselves. In our paper, we present a combination of two stratifications of reflections, which were developed recently in works by other authors. The paper outlines our model and exemplifies its use on the teaching of mathematical models and modelling. The model is preliminary in the sense that it is meant to be tailored by the teacher to fit with the actual mathematical subject in case.

Theme 5

Play and learn with GCompris

Maria Bako, Laszlo Aszalos

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The Commodores, ZX-Spectrums, TRS-80-s and several other home computers appeared in the eighties. It was possible for almost each student to use a computer at home. These machines are advertised as ideal tools for both education and entertainment. In the past, the children who had computers usually used them as diversion. Several home-made educational software was produced, but the quality was low, and only a few people knew about this kind of programs. Nowadays, the children of eighties have grown up, and their children are every-day computer users. They would only like to play with computer programs, but parents

tend to teach them. In this paper we show elements of freeware computer game GCompris, which can help to improve several abilities at the ages of 2–10. We focus on logical-mathematical activities. We show how the ability of counting, recognition of numbers and shapes, logical problem solving, orientation, mirroring, etc. can be improved.

Theme 3

Quantification et variation en mathématiques: Perspectives didactiques issue de la lecture d'un texte de Bolzano

Thomas Barrier

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L'apprentissage de la manipulation conforme des quantificateurs est une difficulté importante du début de l'enseignement supérieur. Nous montrerons, à travers l'étude d'une preuve de Bernard Bolzano (1817a) que cette difficulté a une certaine consistance épistémologique. Pour des raisons d'ordre philosophique, Bolzano est particulièrement concerné par la rigueur de son travail mathématique. Pourtant la preuve que nous étudions contient ce que l'on interprète aujourd'hui comme une erreur. Selon nous, celle-ci est révélatrice d'une véritable difficulté épistémologique concernant les notions de quantification et de variation. Alors que d'un point de vue moderne en mathématiques la notion de variation d'une variable est une notion métaphorique – une variable n'est pas à proprement parler quelque chose qui varie mais plutôt un symbole auquel on peut substituer des objets d'un domaine particulier à travers les règles de la quantification dites d'instantiation universelle ou existentielle – du point de vue de Bolzano, une limite est quelque chose qui est susceptible de varier ou non. Nous utiliserons ensuite un extrait de dialogue entre élèves de première année d'école d'ingénieur pour montrer que cette confusion entre variation et quantification est également présente dans les classes. Notre hypothèse est que ces pratiques permettent aux étudiants de combler l'absence d'un enseignement explicite de la quantification.

¹Laboratoire d'Etude du Phénomène Scientifique.

Theme 3

Concours d'entrée à l'université en Turquie et ses effets négatifs sur les apprentissages des élèves de seconde

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En Turquie, la sélection d'entrée à l'université occupe une place très importante. De plus, ce concours est très sélectif, il comprend des questions à choix multiples et demande des préparations consistantes pour s'entraîner intensivement à des questions types comme celles du concours. Le but de ce travail est d'étudier le concours national d'entrée à l'université en Turquie à partir de quelques notions principales en didactique des mathématiques par exemple dialectique outil/objet, changement des cadres et de montrer les effets négatifs de l'enseignement destinés à préparer les élèves à ce concours sur les apprentissages des élèves. Pour ce faire, nous avons analysé les thèmes du concours de l'année 1970 jusqu'à présent et passé un questionnaire-élève sur fonctions au niveau de seconde. En ce qui concerne les résultats les plus importants de la recherche, les questions du concours contiennent en général des applications directes du cours et aucune d'entre elles ne demande de mettre en œuvre le fonctionnement outil des fonctions. De plus elles sont très pauvres géométriquement mais très riches algébriquement. Par ailleurs, la motivation forte au concours a de mauvaises influences sur les apprentissages des élèves et ainsi elle appauvrit et donne naissance à des nouvelles erreurs.

Theme 3

Le théorème fondamental de l'arithmétique: une approche historique et didactique

Véronique Battie

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Dans le cadre de nos recherches sur le raisonnement en arithmétique, nous avons proposé aux élèves d'une classe de terminale scientifique (18 ans, grade 12) de démontrer à partir de textes historiques l'existence et l'unicité d'une décomposition en produit de nombres premiers pour tout entier plus grand que 1 (théorème fondamental de l'arithmétique). Les discussions qui eurent lieu au sein de deux groupes d'élèves ont été enregistrées. Nous avons analysé leurs recherches en termes de dimensions organisatrice et opératoire : la *dimension organisatrice* s'identifie au raisonnement global qui organise et structure les différentes étapes et la *dimension opératoire* correspond à tout ce qui relève des manipulations calculatoires (au sens le plus large) opérées sur les objets en jeu et permettant la mise en oeuvre des différentes étapes de la dimension organisatrice. Dans cette communication, nous explicitons dans un premier temps ce que représentent ces deux dimensions dans l'exemple étudié ainsi que leurs interactions. Nous présentons ensuite les résultats de l'analyse de la recherche des élèves qui nous a permis en particulier de situer leurs difficultés lors de l'écriture de la démonstration demandée.

¹Laboratoire d'Etude du Phénomène Scientifique.

Theme 5

The use of manipulatives in the teaching of mathematics as expressed in teachers' beliefs

Athinos Constantinides, Nikos Georgiou, Marianna Miltiadou

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The present study examined the use of manipulatives at the teaching of mathematics and which factors likely they influence it. In particular, it was examined how much the years of service and the sex of teachers influence the use of manipulatives, which manipulatives the teachers usually use for the teaching of concrete significances, for which aim, what they would want to be included in a new curriculum and various attitudes for teaching with the manipulatives. According to the results, the use of manipulatives does not appear to be proportional to the age and the sex. Through the research emerges intensely the need of creation of new curriculum for the mathematics, which is supported by the technology and the computational programs, which create dynamic environment of learning.

Theme 5

About the role of real-world knowledge and experiences in mathematical modelling

Cinzia Bonotto

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In this paper we will discuss some teaching experiments, which consist of a series of classroom activities in upper elementary school, using suitable cultural artefacts, interactive teaching methods and new socio-mathematics norms in order to create a substantially modified teaching/learning environment. Through these studies we can show how an extensive use of suitable cultural artifacts, with their incorporated mathematics, can play a fundamental role in bringing students' out-of-school reasoning and experiences into play, by creating a new tension between school mathematics and real world with its incorporated mathematics. The focus is on fostering a mindful approach toward realistic mathematical modelling, problem solving and also a problem-posing attitude. We feel that it is not only possible but also desirable even at the primary school level to introduce early on fundamental ideas about modelling. We also argue for modelling as a means of recognizing the potential of mathematics as a critical tool to interpret and understand reality, the communities children live in, and society in general. An important aim for compulsory education should be to teach students to interpret critically the reality they live in and understand its codes and messages so as not to be excluded or misled.

Theme 2

Definitions in geometry in a South African textbook

Lynn Bowie

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In common with curricula in many other countries, the geometry section of the curriculum in South Africa plays a dual role. Firstly, it is intended to help students develop spatial reasoning and visualisation skills. Secondly, it provides the major context in which students encounter mathematical reasoning and proof. In this paper I will look at how this dual role affects the way geometry, and in particular geometrical definitions, come to be constituted in a key South African textbook for grade 10 learners. In my analysis of the textbook I draw on the work of Houdement and Kuzniak on the three paradigms of geometry: natural geometry, natural axiomatic geometry and formalist axiomatic geometry. I discuss the way the textbook mixes the natural paradigm and natural axiomatic paradigm and the effect this has on the way the notion of definition in geometry is developed.

Theme 1

A pathway for algebraic thinking

Murray S. Britt, Kathryn C. Irwin

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New Zealand has developed a professional development project designed to support teachers in their teaching of numeracy. A major focus of this Numeracy Project as it is known has been the development of a range of sensible strategies for carrying out operations in arithmetic mentally. The attention that the project has given to operational strategies in arithmetic has led to an increased awareness that such strategies are underpinned by structures that are algebraic in nature. In this paper, we discuss the results of three studies that provide growing evidence that working successfully with mental operational strategies involves an awareness of generality that we argue implies engagement in algebraic thinking. We draw particular attention to the results of one of these studies in which students from eight schools in two New Zealand cities were given an algebraic thinking test at the end of each of three years. Results from this test, when compared with scores on a Numeracy Project diagnostic test to determine progress in applying operational strategies, revealed that the students who had developed advanced mental strategies for dealing with additive, multiplicative and proportional operations were the students who were capable of making full use of the symbols of algebra. The findings from these three studies taken together have led to a proposal for a 'pathway for algebraic thinking' accessible to all.

Theme 2

Can any definition of a triangle be considered the best one?

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This paper deals with an important item in Mathematics Education: mathematical definitions. In particular, it focuses on the definition of a triangle, an only seemingly "simple" mathematical object. In order to do this, we discuss the main following points: 1) the distinction between the formal mathematical concept of definition and the use of this word in classroom practice; 2) some specific problems concerning geometry, along with a comparison of many different definitions of a triangle and of different curricular choices; 3) a possible impact of modern technologies on the definition of triangle. Finally, we emphasize the necessity for a teacher to employ a balanced mix of intuition and formalism both in compulsory and in upper school.

Theme 2

The influence of a cultural tool on approaching a problem from the history: solving a geometry problem on graph paper

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This paper reports research findings on the influence that a graph paper had on children's performance in carrying out a geometry task, known in the history of mathematics as duplicating the square, i.e. the drawing of a square whose area is twice that of a given one. Three different formats of graph paper were available to the children testing the hypothesis that differing rulings on the graph paper may structure in different ways the practical activity and consequently influence their problem solving approaches. A qualitative analysis of children's responses to the task identifies differing strategies employed, which may be considered as directly depended on the different visual background provided by the paper used.

Theme 1

Future teachers facing proof problems: Games of interpretation, anticipating thought and coordination between verbal and algebraic register¹

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This contribution is part of a wider work, involving pre- and in-service teachers, aimed at making teachers aware of the importance of developing in students an effective symbol sense. In this perspective, a fundamental part of our work with teachers consists of activities of proof in elementary number theory. At the same time we are interested in analysing the use and the role of algebraic language in the development of such proofs. In this paper we present an initial analysis and classification of trainees' behaviours in facing the proof of a conjecture that arises from their exploration of a proposed numerical situation. The analysis of teacher protocols has been conducted by reference to particular interpretative keys that reflect our thinking about the abilities required for the deployment of algebraic language in proofs. They are: the application of specific conceptual frames, the games of interpretation between different frames, anticipatory thoughts, the use of conversions and treatments and coordination between different registers of representation. From an educational point of view this analysis can be used as a methodological tool to both help teachers identify their own difficulties in algebra and to promote communication with and amongst them.

¹Work carried out in the European Project 'Transforming Mathematics Education through Teaching-Research Methodology'.

Theme 2

Les paramètres didactiques cruciaux pour comprendre l'intégration de l'expérimental dans la pratique et l'enseignement de la géométrie: Exemplification grâce à Cabri 2 Plus et Cabri 3d

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En France, l'utilisation des TICE dans l'enseignement des mathématiques est maintenant souhaitée par l'institution car on pense qu'elle devrait permettre une approche plus expérimentale des mathématiques dans un pays à tradition fortement rationaliste. Cet article met l'accent sur certaines des connaissances ne faisant pas partie de la formation actuelle des enseignants et que je considère comme des paramètres cruciaux pour maîtriser un enseignement s'appuyant sur une démarche expérimentale de découverte médiée par l'environnement de géométrie dynamique Cabri. Ces paramètres cruciaux concernent la notion de figure (qui est une spécification), la notion d'expérimentation (qui est une pourvoyeuse de données et

qui est, soit générative soit validative), les appréhensions figurales avec leurs valeurs heuristiques en particulier en environnement informatique. La connaissance d'une décomposition formelle de la démarche expérimentale de découverte (en étapes pré et post conjecture) ainsi que des praxéologies G1 et G2 Informatiques mises en évidence dans mon travail de thèse (mettant l'accent sur les techniques de validation spécifiques en environnement de géométrie dynamique) fait aussi partie de ces paramètres tout comme la preuve expérimentale qui est une partie fondamentale d'une telle démarche en amont de la démonstration formelle, qu'elle ne remplace pas mais qu'elle doit préparer.

Theme 3

The mathematical education of women in Greece (19th - early 20th century)

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The education of women in Greek territory, namely Greek communities of the Ottoman Empire and Greek state, first appears at the beginning of the 19th century with the operation of a small number of non-systematically organized schools. Women's systematic education begins in 1830s and 1840s for the state and communities respectively, with the operation of a) communal, self-sustained and "national" educational institutions in the communities' territory, b) state institutions – under the responsibility of municipalities – in the Greek state, and c) private schools in both of the aforementioned spaces of Hellenism.

Women's education constitutes a distinct educational system, separate from that of men, not only as regards its structure, the courses provided and the duration of studies, but also as regards the school curricula (taught subjects and their content). This differentiation results from the fact that female education was organized on the basis of the "different nature" and the "different social destination" of women; gendered perceptions that were further reinforced during 19th century by the socio-biological theories (Social Darwinism), that had a great impact on Greek scholars and pedagogues.

With respect to the mathematic education of women, judging by the Maths teaching goals and textbooks, the school curricula, the timetables and the educators' published opinions with regard to the role of Maths and Sciences in female's education, it emerges that their Maths education is limited at both, range and type of content; for example, Geometry and Algebra is not taught at all, whereas girls were taught from Arithmetic only the four basic skills, i.e. addition, subtraction, multiplication, and division, and only in exceptional cases women's instruction included the teaching of decimal numbers. Also, their mathematical education was practically oriented towards their future social destination and the needs that women would be called to meet in private sphere as wives, mothers and house-keepers. The shrunken provision of mathematical knowledge was founded on 'scientific' stances of the era concerning the limited intellectual capabilities of women, along with the damage that intellectual strain causes to their reproductive (!) ability.

This overall attitude towards the mathematical education of women is differentiated to some extent on the basis of: a) the time period of the schools' operation; for instance, during the first period of women's education, namely 1840-1860, their mathematical education was wider and more enriched due to the influence of innovatory principles of Greek Enlightenment, b) the geographical region (in the area of Smyrna the competitive functioning of Greek schools towards the missionary and western schools lead to more radical educational attitudes), and c) the administrative type of schools, which was related to their class orientation; in the self-sustained schools, for instance, where girls of the superior social class studied, not only the selection of the provided knowledge but also the teaching methodology and the media of instruction were more innovative and wider.

The present paper is based mainly on the study of primary sources (Minutes of teachers associations, Rules and Regulations of schools, timetables and curricula, examination schedules, viewpoints held by scholars and educators of the above period) and is built by a combination of the historical-comparative and the interpretative methods.

Theme 3

Les découvertes Keralaises sur les séries trigonométriques et leurs justifications dans le *tantrasangraha* (1500 d.n.e.) et ses commentaires.

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Pendant plusieurs siècles, la tradition d'*Aryabhata* (astronome-mathématicien des V-VI^e siècles d.n.è.) a été préservée dans le sud de l'Inde, plus particulièrement au Kerala. Soucieux d'appliquer les paradigmes classiques pour le mouvement des planètes, les astronomes kéralais (comme *Jyesthadeva* floruit entre 1500 et 1575, élève, comme *Nilakantha*, né en 1444, de *Damodara*, fils de *Paramesvara*), constatant que leurs calculs ne correspondaient pas à leurs observations, ont changé les constantes et paramètres de leurs modèles. Ces astronomes, jeunes et vieux, vivaient collectivement dans des *illam* (mot malayalam désignant la maison, la résidence), grâce au patronage du roi auquel ils rendaient les services habituels de leur savoir (astrologie, établissement de calendriers, almanachs, etc.). De cette manière, les connaissances circulaient librement et s'enrichissaient constamment.

Une particularité de ces cercles de lettrés kéralais est la production de textes de base sur certains sujets particuliers (au contraire du reste de l'Inde où l'on trouve plutôt des sommes), ensuite développés et commentés. Leurs travaux sont surtout remarquables dans le domaine de la trigonométrie, dont l'origine est le désir d'améliorer les approximations de π , les tables de différences de sinus et l'évaluation des sinus d'angles non tabulés, utilisés en astronomie. Dans ce sujet, ils devancèrent de près de deux siècles les découvertes du XVII^e siècle européen.

C'est à la description de certains de ces travaux, exposés particulièrement dans le *Tantrasangraha* de *Nilakantha* (v. 1500 d.n.è.) et dans la *Yuktibhasa*, son commentaire anonyme du XVI^e siècle attribué à *Jyesthadeva*, et à la recherche de leur présence dans d'autres ouvrages kéralais de la même époque, que nous consacrerons cet exposé. Nous verrons que, contrairement à ce qu'on observe dans les autres ouvrages mathématiques indiens, y compris chez *Aryabhata*, qui énonce simplement les formules, ou chez son commentateur *Bhaskara I*, qui les illustre par des exemples, les astronomes-mathématiciens kéralais explicitent les raisonnements heuristiques qui mènent aux « formules trigonométriques ». On peut considérer ces raisonnements comme la manifestation, unique dans l'histoire de l'Inde pré-coloniale, d'un souci d'explicitation, sinon de prouver des résultats, que les historiens des mathématiques ont trop tendance à considérer comme empiriques.

Theme 2

Teaching geometry in secondary school using van Hiele's theory and ICT in the context of cooperative learning

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According to national and international research, pupils, when taught geometry using the traditional instructional methods find difficult a) to understand geometric concepts, and b) to successfully write proofs. In this paper we propose a teaching method for geometry in the Lyceum. This method stems from the combination of the van Hiele's phases of instruction with the Cognitive Apprenticeship model in the context of modern ICT. Research was conducted in a sample of 250 pupils, aged 15-16 years old, in order to investigate the performance of these pupils in geometry. In this paper we present the results of this research.

Theme 2

Reinstatement of vectors in lower secondary education: some didactical suggestions and comments

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The vector is a multidimensional and multilevel concept, connected with conceptual and learning difficulties that have been identified empirically. Moreover, it has been marginal in Mathematics of Lower Secondary Education for a long time. As a result, students form partial conceptions, mostly under the influence of physics courses.

The new national curriculum in Greece places the concept of vector back in Mathematics of the second year of Lower Secondary School, obviously to be used as a mathematical tool in Physics. This is a rather interesting effort, regarding the choice of teaching time, as well as, its content. However, for simplicity reasons or economy in presentation, some crucial points are either omitted, or presented in an insufficient, or improper way.

In our work, we comment on these crucial points and suggest some alternative ways of presentation.

Theme 3

Learning mathematics: a naturalistic view

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Beliefs concerning the general status and quality of mathematics and also those indicating, what mathematics is all about and its accessibility and comprehensibility often turn out to have negative and detrimental effects on learners' attitude towards mathematics. For instance: mathematics is viewed as an abstract discipline, which is mostly interpreted as inaccessible for the average student; or, mathematics is too difficult for girls; or, doesn't make any sense operating with meaningless symbols. I will discuss a possibility to counter these related beliefs, which in my view are not compatible with the reality of actual and observable mathematical activity. The theoretical background is formed by the notions of diagram and diagrammatic reasoning developed by Ch.S. Peirce within his semiotics. These offer the opportunity to consider the very diagrams, i.e. written inscriptions together with a system of formation and transformation rules, as the objects and means of mathematical activities. In this way, mathematical activities receive a very concrete quality, becoming observable and imitable, as well as turning into a kind of social practice comparable with that of crafts. In these activities one designs diagrams, one investigates their properties, states invariant relations as theorems and makes a great variety of referential interpretations of the diagrams. I will present the central features of the theory, give many examples and guard this view against misinterpretations, e.g. as a kind of behaviourism or as a kind of formalism.

Theme 3

Cypris Stephanos and the Erlanger Programme

Dedicated to Prof. Hans Wussing in his 80th year

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At the turn of 20th century Cypris Stephanos (1857-1917) Hermite's pupil was a very well known mathematician in Europe. His name appeared in the editorial board of many scientific journals and he also was the Greek representative on two important committees: Répertoire Bibliographique des Sciences mathématiques (president H. Poincaré) and International Committee of Mathematical Education (president

F. Klein). As a secretary of the French Mathematical Society and through his research (invariant theory and geometry) he had the opportunity to gain international esteem. In this paper we will discuss Stephanos' attempt to publish in French his translation of Erlanger Programm which unfortunately was never realized despite Poincaré's support.

Theme 1

Students with low performance and the multiplication operation

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This study focuses on the exploration of the ways in which two Y5 students with learning difficulties in mathematics conceive multiplication as an operation. For this purpose we analyse the strategies and techniques that the two students used to find the results of a number of multiplications as well as the representations of these multiplications. The students used a variety of strategies for their mental calculations while they produced poorly conceptualized representations for multiplications with one factor bigger than ten. It seemed that the two students did not develop their techniques to become structures and tools for the learning of the tables. Supporting students with learning difficulties in multiplication requires knowledge of the specific difficulties that those students face in conceptualising the concept itself or properties that facilitate understanding and development of the concept.

Theme 4

Fundamental Ideas and Basic Beliefs in Stochastics

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The concept of fundamental ideas has been introduced by Jerome S. Bruner, an American psychologist, in 1960 ("The process of education"). It means that education in a special subject should follow the main lines, which the related science pretends. First of all, this approach concerns not only the content of education, but also the attitude, which is characteristic of doing mathematics, for instance. So in this sense mathematics education should be a non-biased copy of the science of mathematics.

Of course the level in education must be different from that one in science. This means not a restriction, but a challenge for the didactics of mathematics to identify typical contents and methods in mathematics, or any other subject, for its education. It is important to mention that certain themes are discussed over and over again at increasing levels (spiral principle) so that there is a chance to achieve sustainability.

Basic beliefs are a second concept in the didactics of mathematics to achieve sustainability of mathematics education. They were introduced by Rudolf vom Hofe, a German mathematics educator, in 1995. In opposition to the fundamental ideas, now the probands, who are doing mathematics respectively are confronted with it are focused. We have to distinguish between normative and descriptive categories. The first ones, which characterise the teachers' input, are – similar to the fundamental ideas – orientated to logical connections and also include didactical elements; for instance experiences about the knowledge and other circumstances of a certain age cohort. The second ones describe the individual pupils' beliefs concerning cognitive contents, as well as, affective attitudes.

In this talk, on the one hand, I will try to present some ideas about fundamental ideas in stochastics education; on the other hand, based on my experiences as a student teachers' educator I would like to enumerate some big points of a lecture course in stochastics at the university level.

Especially there is one aspect I will stress in order to explain how mighty the conception of fundamental ideas can be: the possibilities to connect stochastic themes with topics, methods, and ideas from other mathematical fields.

Theme 4

Instructional design and research design principles in probability

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We created a basic probability unit based on design principles such as opportunities for hypothesizing, facing conflicts, arguing, and experiencing inverse questions. Above all, the unit has a hierarchical structure which enables: a process of constructing knowledge, research observation of this process and opportunities for students to cooperate, in spite of their different levels of knowledge and their different abilities.

In this paper, we will describe the "design principles" and demonstrate them by means of relevant problem situations from the probability unit. We will show how the principles serve both, instruction as well as research, by exhibiting 8-grade students' responses in the research to the above problem situations, thus demonstrating whether and how their learning was influenced by the principles.

Theme 1

Characteristics of 11-year-old students in connection with the estimation and control criteria of the outcome of arithmetic operations

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Although the construction and the use of control and estimation criteria is a very important process in mathematic education, there is a deficit of didactic organisation with regard to the teaching of the particular subject. On the basis of these ascertainment: a) we designed and gave a test to sixth grade students and b) we designed and made a didactic intervention on sixth grade students of a Greek Primary school (experimental group) in order to investigate their abilities and their behaviours with regard to the control and estimation criteria of the order of magnitude of arithmetic operation. In this paper we try to adumbrate the profile of the students of the experimental group after the application of the didactic intervention taking into account their performance in other mathematic areas.

Theme 1

Scheme and its generic models

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Schema is understood as the memory structure that incorporates clusters of information relevant to comprehension. It gets embedded in a person's mind by repeated "stay" in a certain kind of environment (one's house, school, shopping centre). Scheme-oriented mathematics education is described and illustrated by means of adding and subtracting. This paper surveys the experience with the implementation of this teaching method in teacher-training facilities.

Theme 2

What does it mean for a teacher to have a "scientific approach"? A reflection

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Assumptions about the construction and the transmission of knowledge and about the nature of

mathematics always underlie any teaching practice, even if often unconsciously. To make explicit such assumptions and to enrich on a theoretical basis the naïve models of natural knowledge dynamics and of knowledge transmission constitute a first step in order to make didactic practice a more and more “scientific” enterprise. In this paper we present a piece of research experience made by a teacher, involving the study of some new neuroscience results, and analyse its outcomes in her didactic practice. The teacher’s ability to rely on a theoretical reference frame appears in several directions: in the choice of the global goals in mathematics education, in the interpretation of learners’ cognitive behaviours, in the design of didactic interventions, in the assessment, and so on. To support our argumentations, we present some experimental data concerning a long-term path with 14 year-old pupils.

Theme 3

Evaluating a teaching module on the early history of error correcting codes

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This paper falls within the area of empirical research in the field of using history in mathematics education. More precisely, it discusses a teaching module on the early history of error correcting codes, which was implemented in a Danish upper secondary mathematics class in the summer of 2007. As part of this module the students were to write several essays illuminating different aspects of the early history of error correcting codes. The research questions of the paper concern the way in which the students were able to discuss meta-perspective issues of this history and if these discussions were anchored in the taught and acquired mathematics of error correcting codes.

Theme 1&2

Findings from a diagnostic test given to students in the 1st year of the Greek Lyceum in 2006-07

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The present work is a report on findings of empirical research, which attempted to estimate by means of a Questionnaire the general picture of the effective naïve knowledge of elementary Mathematics of pupils who entered the school year 2006-07 in the second phase of the Secondary Education in Greece. The Questionnaire has been created by the author of this empirical research. The content of the questions stems almost mainly from the mathematical textbook that the subjects used in classroom and for homework during the previous school year (2005-06). The conjecture of the empirical research is that the subjects in question show at the beginning of the school year considerable amount of knowledge gaps and inefficiencies which have to be dealt with, if progress is meant to be made in the field of Mathematics in Secondary Education. The information gathered through the Questionnaire constitutes a strong indication that the conjecture lays on a real basis, although it cannot be considered as proof of facts.

Theme 1&2

Reception of the reformed mathematics curriculum by the nursery teachers of public education in Greece

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This paper presents nursery teachers’ perceptions about the reformed curriculum for nursery education. It attempts to explore teachers’ perceptions about their self-efficacy in teaching math concepts and the efficacy of teacher training program for teaching such concepts. It also attempts to explore teachers’ perceptions about the applicability of the reformed curriculum and the degree of its implementation by teachers in their daily teaching practice. Results analysis indicated that teachers’ perceptions about their

self-efficacy have an important role in the application of the reformed curriculum.

Theme 3

Modeling tasks for developing geometry teachers' spatial, epistemological, and pedagogical knowledge

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What is the nature of mathematical knowledge for secondary teaching? International researchers agree that secondary mathematics teacher preparation must address the procedural, rule-bound nature of prospective teachers' understanding of many secondary mathematics topics. No longer is it assumed that advanced university-level mathematics coursework is adequate preparation to teach secondary mathematics. Aspiring teachers majoring in mathematics often must de-construct their rule-bound understanding of familiar school mathematics topics and re-construct it into a more dynamic and penetrating understanding of concepts, algorithms, methods of inquiry, and disciplinary perspectives. The challenge for teacher educators is to generate critical analysis, reflection, and penetrating discussion about topics that prospective teachers think they know. Video cases of students' thinking are known to stimulate provocative mathematical discussions among teachers. In this paper, I propose *manipulative modeling tasks* as another knowledge-eliciting and knowledge-promoting cognitive tool for mathematics teacher preparation programs. The tasks reported here focus on non-Euclidean geometry.

Theme 3

On composing multiple choice tasks, thought experimentation and algebra teachers' knowledge base

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This article deals with characterizing the knowledge base of 55 in-service high school algebra teachers, with particular focus on the teachers' knowledge of students' epistemology. We report the data collected during three workshops, in which the teachers were asked to compose optional (wrong, but plausible for some students) answers to the question related to exploration of functions. The workshops were videotaped; three out of 13 discussions in small groups were audiotaped and transcribed; all the written work was collected. The resulting answer options composed by the teachers and their problem posing discourse were analyzed inductively, based on the predefined categories taken from Schulman's definition of teachers' knowledge base. We argue that the teachers' performance bears characteristics of thought experimentation. The teachers are found to be capable of constructing multi-step imaginary scenarios of students' erroneous thinking based on various rational reasons. The results also show that less experienced teachers tend to project their own learning experiences on their students, whereas more experienced teachers model the students' thinking by reflecting on their teaching experience.

Theme 2

TRICK

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In the presentation the project week for 16-year-old students with the title "Trick" is introduced. It describes how to connect mathematics with psychology, biology and art by studying optical illusions, impossible objects, Möbius strip, reversible figures, and the fourth dimension.

Theme 5

Teaching linear dynamic systems in elementary school, using simulations: Sensitive dependency on the initial conditions

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Since many years, the study of non-linear dynamic systems constitutes a new research area in contemporary Mathematics and Natural Sciences. The systemic theories, such as complexity theory and chaos theory, have an important contribution in the way humans conceptualise ‘reality.’ What we teach in schools about the non-linear dynamic systems, their properties, and the implications of any human intervention influences the way our students construct meaning relating to them, and how these future citizens will deal with any issues relating to them.

The present study attempted to investigate how a sample of 16 11-12 years-old children from a primary Greek school conceptualize how the functioning of linear and non-linear dynamic systems depends on initial conditions. More specifically, the study put forward the following questions:

1. To what extent can sixth-grade students correctly distinguish the outcomes of non-linear and linear dynamic systems when only the initial are different?
2. Are sixth-grade students able to transfer their understanding about the dependency of these outcomes on the initial conditions to other similar situations?

Initially, students were asked to individually answer questions about the functioning of linear and non-linear dynamic systems when only the initial conditions were different. Then, the students were assigned the task to work with simulated linear and non-linear dynamic systems, where it was possible to continuously change the initial conditions and repeatedly repeat the functioning of linear and non-linear systems and investigate questions such as “what would happen if...” The basic principle behind the simulations of the functioning of non-linear dynamic systems was closely focused on representing their complexity. This complexity was attributed to a small number of mutually interdependent variables that were included in the simulations. Students were involved in the research procedures using the software “Game of Life” and “POLLAPLAN” for investigating the functioning of either non-linear or linear dynamic systems, respectively. Interviews were finally conducted with a sample of students representing the different categories of responses that emerged from students initial responses to the questions that were put forward. The interviews constituted an application of a “teaching experiment,” where the main purpose was to model students’ understandings about learning and the process of students’ learning. This modelling process is still in progress.

Theme 4

Didactical investigation of a simple physical model of moving particles for improving the understanding of variance in statistics

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Didactical research points out that variance and standard deviation are difficult concepts for many students who follow introductory courses on Statistics. Our previous research work points out that in the context of usual examples of statistical data referring to everyday-life situations (and more generally to situations related to social phenomena) the understanding of variance presents specific and important conceptual difficulties. Moreover, this research work point out that during the introductory teaching of the concept it is also important to present to the students situations in the context of which: variance has a clear and easily understandable meaning and becomes evident that it is a basic measure of dispersion. A didactically oriented historical and epistemological study has allowed us to determine some simple physical models that satisfy the aforementioned characteristics and their didactical use is possible to facilitate students’ understanding of variance. A basic model of this kind is the model of moving particles whose didactical use was investigated within an experimental course of introductory Statistics that we have been realized

with students of the Department of Education (prospective primary school teachers). In the present paper, some elements of this investigation are analyzed.

Theme 5

The notion of proportion in vocational activity and its relation to school experience: a case study with Cypriot bakers from Lemessos province

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In the past few years, several studies were realised in the field of mathematic education, with regard to the use of mathematics in various professions, such as nurses, carpenters, civil engineers etc. Almost all of these studies led to the same conclusion: most adults use mathematics, in order to comprehend situations around them, in such ways that differ radically from those who are taught in school. In the present study, we try to make a first investigation in the proportional reasoning of bakers. Specifically, we examine: their opinions regarding the usefulness of school mathematics, the relation of their knowledge in proportions with what they use in their work and the strategies of proportional reasoning that they use in their work. The results show that the bakers of our sample believe that in their work they use strategies of solving problems involving proportionalities that they learned at school and they use mainly reduction to unit fractions. They have a positive attitude towards mathematics and most of them believe that they need school mathematics and mathematics in general, in their life and profession.

Theme 1

The use of algebra tiles in the teaching and learning of algebraic concepts

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In this work, which is part of a wider research, we study some of the possible ways by which a special type of manipulatives, namely, algebra tiles, influenced the teaching and learning of algebraic notions, like a perfect square, the development of the 2nd degree binomial, the difference of squares etc

Theme 2

Dynamic implementation of Moulton's plan (non-Desarguesian geometry)

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As it is well known, David Hilbert was the first to realise the importance of Desargues' theorem for an axiomatic approach to plane geometry that makes no use of 3D-space. An example of geometry that realizes all Hilbert axioms except Desargues' theorem, was proposed by Moulton in 1902, since then known as Moulton's plane.

Up until recently, the different striking achievements in dynamical geometry did not give the possibility to model Moulton's plan dynamically because of the difficulties in using it's straight lines and segments as one object. This is no longer the case and the exploration of Moulton's plan can be done very rapidly once the implementation has been done.

This exploration is of great interest, notably from a didactic point of view, in the context of pre-service teachers education.

- Firstly, it perfectly illustrates the ever-present question of using motion in geometry and reveals the concept of Hilbert's congruence when no usual motion is possible.
- The properties of orthogonality are particularly new: Being neither hyperbolic, nor elliptic geometry, Moulton's plane partly shares properties between these two geometries. In this way, a straight line cannot have a perpendicular issued from a given point, but it can have two, issued

from another point, and if it has two, it doesn't have an infinite number as in the case of elliptic geometry.

- The same thing holds for angles: the sum of two angles of the same triangle can be greater, or smaller than two right angles, depending on the case.
- And lastly, the distance is not a metric (in the sense of a metric space), which creates certain peculiarities (perpendicular bisector, angle bisector).

In this presentation, we will explore different proprieties of Moulton's plane, guided by what can be done with these properties:

- In courses on general non-Euclidian geometry.

In mathematical investigations with pre-service teachers.

Theme 4

EarlyStatistics: an online professional development program for European teachers of statistics

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This article provides an overview of EarlyStatistics, a 3-year program funded by the European Union under the Socrates-Comenius action. EarlyStatistics aims to enhance the teaching and learning of statistics in European schools by harnessing the power of the Internet to provide European teachers with access to a wide array of colleagues, discussions, and resources eluding them in their workplace. The project will offer a high-quality online professional development course in statistics education to elementary and middle school mathematics teachers around Europe. It will facilitate intercultural collaboration of teachers using contemporary technological and educational tools and exemplary web-based materials and resources. Long-term sustainability will be assured through support of multilingual interfaces and online services for the accumulation of collective knowledge from end-users. *An online* knowledge base will offer access to usable and validated pedagogical models, didactic approaches, and innovative instructional materials, resulting in a complete and flexible teacher professional development program.

Theme 1

The multicultural dimension of multiplication: attitudes and perceptions of Cypriot educators about its use in teaching mathematics

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In recent years, there has been intense debate about the role of the integration of the history of mathematics in mathematics education. Educators are the key to its "successful" integration. In this paper, we study the attitudes and the perceptions of educators at primary level in Cyprus towards the integration of the history of multiplication in the teaching of mathematics, as well as the educators' personal characteristics that affect these attitudes. Through the quantitative and qualitative analysis of the answers of the educators in the sample, it has been found that they have a positive attitude towards the integration of the history of multiplication in its teaching, regardless of their sex, further studies and knowledge of the subject. However, it has also been found that the educators in the sample have little knowledge of the educational value of the integration of the multicultural dimension of the operation of multiplication in its teaching.

Theme 4

The didactics of statistics in compulsory education

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For any educational subject a contextual setting should be presented in regards to desirable targets, concepts, knowledge and skills that are to be learned and respectively assessed. Common targets that could be implemented to different levels and contexts of education should be recognized. In this presentation we comment in general to a similar context regarding statistics in basic education (reforms, educational targets, teaching model, content and didactic approaches).

Theme 3

The research on metacognition and the learning of mathematics: facts and perspectives

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Learning procedure has abandoned its traditional style and became more interactive with five factors: the subject, the society, the family, the school, and the learning structure. This fact has given more space in the role of metacognition.

In this paper we propose the classification in four categories of some researches about metacognition in the learning of mathematics and we try to up heave the posed questions.

- Research about the role of metacognition in the learning and didactics of mathematics
- Research about the relationship between metacognitive abilities and cognitive procedures
- Research about the function of metacognition
- Research about the using of other learning environments that help the development of metacognitive abilities in mathematics.

Theme 3

One, zero and infinity: three cases of historical constructions in the context of teachers' training

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One, zero and infinity consist three cases of historical construction in the universe of numbers. Jean Piaget believes that when searching the first (cardinal) number in objects, the series of numbers starts from number one. When did we historically have the appearance (construction) of zero and when did we put it in the place of the first number? Which were the arguments of ancient Greeks to exclude the “one” from the field of numbers? In which way, when and in the context of which mathematical tradition did “one” become a number? Which is the cardinal number of the biggest set and in which way is this number related to the notion of infinity? The questions we have tried to work with the students are practically associated with the answer to the question, which is the smallest and the biggest natural number (cardinal) they know and if different mathematical traditions have this knowledge.

The analysis of these questions is part of our effort to give answers to the question which is more and more often asked to the teachers of Primary Education in the context of the dimension of knowledge for knowledge for action and is the following: which should the initial and the continued training of teachers be aiming to the gradual introduction of the cultural and historical dimension of teaching of mathematics? The application of a series of projects in the context of a semester course under the title “History of

mathematics and mathematics education” in the Department of Primary Education of the University of Western Macedonia tries to give some explanations to the difficulties we found which have caused the “one” not to be considered as a number until the intervention of Simon Stevin. We study the notion of zero in the mathematical traditions of Babylonians, Maya and Hindu, a history of three places and we study the passage from the vacuum place to the zero quantity in the context of the Hindu mathematical tradition. In parallel, relatively to the notion of infinity we make use of the paradigm of the known text “Infinity Hotel” of D. Hilbert and we study the notion of infinity in power of Aristotle till the infinity cardinal number of G. Cantor.

Theme 2

Overcoming obstacles in geometry by deploying representations of ICT

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There is evidence from educational research and everyday teaching practice that students have difficulty in understanding geometry. These difficulties seem to appear due to the existence of some obstacles as defined by Bachelard, and referred to by Brousseau. This situation results in students’ poor performance in geometry. In this paper, we claim that teachers have the potential, by deploying dynamical representations of ICT in the teaching-learning process, to help students override some of these obstacles.

Theme 4

Developing statistical reasoning in the early grades: a new role of technology

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This paper focuses on the development of young students’ statistical reasoning skills. It presents findings from a study which examined third grade (8-year-old) students’ interactions with the data-visualization tool TinkerPlots® (Konold & Miller, 2005), a dynamic software specifically designed to meet the learning needs of students in the early grades. This study is part of the EarlyStatistics program, which is funded by the European Union under the Socrates-Comenius action. The data presented in the paper, come from a scenario where third grade students in an urban elementary school in Cyprus conducted a survey about health, nutrition and safety habits of students in their school. Children analyzed their collected data using TinkerPlots® as an investigation tool, and made a presentation of their findings to the whole school. Based on this case study of a group of 8-year-old students, the following questions regarding young learners’ development of informal notions of inference are explored in the article: (i) How do young learners begin to reason about data? (ii) How can the affordances provided by a dynamic learning environment be utilized in the early years of schooling to scaffold and extend students’ statistical reasoning?

Theme 2

The development of students’ geometrical thinking through linking visual active representations

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The paper draws on a didactic experiment conducted in a secondary school mathematics classroom in Greece, which aimed to explore the ways in which students develop geometrical reasoning and rigorous proof through building visual active representations. The students explored cooperatively multiple-page sketches (half) predesigned by the researcher in an open problem/activity using the interaction techniques supported by the Geometer’s Sketchpad dynamic geometry environment, in order to solve and to reason by

rigorous proof. The paper introduces the meanings of Linking Visual Active Representations (LVAR) and Reflective Visual Reaction (RVR). The researcher (S.P) observed the students' actions and thinking processes during the process of investigation and offers a description of these processes. Two distinct themes emerge from the recording of the results: the relationship between: a) the design/building of the activities and the RVR by the students and b) LVAR in the activities and the development of geometric thinking. LVAR, RVR and van Hiele levels are used as descriptors in this analysis.

Theme 1

Exploring the performance of 9 - year - old elementary school students in doing arithmetical operations and problem solving during the initial implementation of the new textbooks and reformed curricula

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This paper presents research findings that show that an important rate of pupils finishing the third grade of elementary school face difficulties to solve written and mental calculations and in problem solving, during the first implementation of new curriculum and the new textbooks of mathematics. The research sample consisted of 36 pupils, 17 of them were boys and 19 girls. They attended in two different classes of two public primary schools of city of Iraklion. Initially, the theoretical background is given, followed by the description of the methodology and the sample that was used. Next results of the research are presented and discussed and the paper concludes with some proposals.

Theme 1

Classroom activities based on the concept of a Pythagorean triplet

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In this presentation, a didactical sequence of 30 activities has been taught, related to the understanding of the concept of a Pythagorean triplet. These activities are of increasing difficulty and have been designed for 15-16 year old high school and senior high school (Lyceum) students in Greece. Most of these activities have been implemented in a real class at the Experimental School of the Aristotelian University of Thessaloniki, Greece.

The major task of this approach is to help the students to connect different concepts in number theory, algebra and metric geometry. Another task is to make students raise questions related to a central concept.

Theme 3

From visual to logical argumentation within intentional designed activity

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The goal of this research is to trace the development of argumentation processes of peers of students working on an activity in geometry, intentionally designed to afford reasoning and argumentation.

The design of the activity and its implementation, integrates several ideas and intentions:

- 1) Designing situations in which contradictions between students' intuitive conjectures and experimentation's outcomes might occur.
- 2) Capitalizing on the potential of Dynamic Geometry (DG) software for identifying geometrical properties through inquiry methods.
- 3) Encouraging interactions between the students in the dyad and observing differences between the characteristics of their initial conjectures, their ways of solving the tasks and the ways they argue about

it.

4) Tracing the effects of such interactions on the students' argumentation processes.

We will show students' transformations from arguments based on visual features and/or on intuition, through measurements and experimentation in the DG environment, to logical arguments including abductive and deductive processes.

We conclude that a proper design of an activity in geometry, should use didactic tools like argumentation, should face contradictions and should use the power of DG software; because they all contribute to a rich and an active construction of knowledge by students.

Theme 5

How to “domesticate” logarithms in school? Giving sense to the logarithmic function

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In this article I would like to explain how math teachers can show students where logarithms are applicable in everyday life and in this way help them to construct a network of multiple interconnected and related concepts that includes the logarithmic function. Once students gain a sense of the logarithmic function, they learn and better remember the topic. I present some activating tasks for students. I have used them in my class after the introduction the logarithmic function as a purely mathematical concept and it was very successful.

Theme 5

Relations between “saying” and “practice” of mathematics teachers when they teach geometry to students aged 12-15

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This presentation is inscribed in the frame of the qualitative research “*The reflective class in Mathematics classrooms of the Third Cycle of General Basic Education (EGB3)*”¹ when geometry contents are treated”. It is focused on what can be called the “*Geometry of the teacher*”², a complex truss that joins *teacher's disciplinary conceptions*, as a result of a specific preparation, *teacher's assessment* as a teaching content and *teacher's performance* when teaching and propitiating reflective processes on students in order to construct mental representations for the comprehension of spatial relations.

We discuss a comparative analysis of the data that were obtained from interviews to four Mathematics teachers whose geometry classes were observed. From this comparison we pretend to establish possible correspondences between the “saying” (what the teachers declare during the interviews about teaching and learning geometry contents) and the “practice” (teachers' behavior and actions during the observed geometry classes). We detail significant aspects detected on declarative perspectives and teaching practices. Then, we explicit relations that were detected between both aspects, and especially different modes of articulation between the teacher's reflections with actual actions implemented in the classes.

¹The Third Cycle of General Basic Education (EGB3) is attended by students aged 12 to 15 in Argentina since 1994.

²The term “*Geometry of the teacher*” was inspired by the article of Halbwachs, published on *Revista de Enseñanza de la Física*, (1985) where he places the “Physics of the teacher” between the Physics of the physician and the Physics of the student.

Theme 5

Stepping and ladder methods in the process of building up an additive triad schema and other mathematical schemas

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Our research indicates that numeracy skills can be cultivated by the process of building up an additive triad as a concept or even as a schema. There is a wide range of semantic models of an additive triad. The most demanding model of all these is that in which all the numbers in the triad function as operators. And these are precisely the models, which tend to be the most neglected ones in the traditional teaching of mathematics, which may in turn have a negative impact on the arithmetic abilities of upper-grade pupils. One such alternative teaching environment - “Stepping”, which has been experimentally tested and proved successful, offers a didactically plausible way of eliminating that serious flaw. The “Stepping” environment is a basis for getting a procedural grasp of positive and later also negative numbers, numerical axis and other basic arithmetic terms. This helps build up the additive triad schema and further mathematic schemas, which would relate to equation systems, divisibility, probability or combinatorics. This is closely followed by the so-called “Ladder” environment, which focuses on the process of conceptualising the additive triad schema.

Theme 5

Writing as a problem solving process and anxiety levels of students with writing difficulties

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Writing of a communicative text is approached by many students as a problem solving process in their effort to compose it. These students take under consideration the goals they want to achieve in the specific context of communication, the kind of text they want to write and the audience they address to. Low-level students – writers and students with writing difficulties approach writing as a tool of evaluation from which negative emotions derive. These students compose a text in a similar way they produce speech and make use of ineffective strategies. Our paper presents a didactic approach according to which the composition of a text is a process similar to the process of problem solving. It also emphasizes its effects in students with writing difficulties in relation to the use of composition models, the quality of writing and the anxiety levels.

Theme 5

Using video cases to promote good practice in the teaching of mathematics

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The article briefly presents mathematics education courses grounded in the analysis of video recordings of mathematics lessons. Some techniques of such analyses are given. My idea of **effective** practice in the teaching of mathematics is presented via seven principles guiding the teacher's action in the classroom. The on-going research project focused on students' ability to react to their own teaching practice and that of others is presented. Its methodology is outlined (research questions, data collection) and an example of the analysis of two pairs' discussions about the video of one mathematics lesson is given. It is shown how such a reactive discussion can be described and evaluated. Tentative conclusions concerning the influence of different types of analyses of videos on the above ability are drawn. Keywords: lesson study, education of mathematics teachers, effective practice in teaching mathematics, analysis of videos.

Theme 5

Teaching mathematics through pupils' projects

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This text describes project work and focuses especially on teaching mathematics at lower grades in the secondary school. Project work is presented as one of the tools of implementing innovative teaching approaches currently promoted in the Czech Republic by a new school reform. The first part deals with the term project and how it is understood by different people. It also underlines main steps that should be followed when teachers and pupils are preparing and running a particular project. The second part is practically oriented. It presents one particular interdisciplinary project (the project was focused on the revision of triangles with 12-year-old pupils), which was organised in a classroom. The process of its preparation, realisation and evaluation is also presented as well as the pros and cons of project work.

Theme 3

Snapshots and pictures from integrating the history of mathematics in the new textbooks of secondary school in Greece

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The new official textbooks for the teaching of mathematics in the Greek Gymnasium (13 to 15 year old students) contain a lot of historical material, which has been included following respective recommendations of the new curricula. Besides the usual historical notes, there are also many activities with historical content aiming obviously to provide teaching tools for better understanding the textbooks' mathematical content.

However, a critical reading of this historical material reveals that the principal reason of its incorporation has been rather the authors' obligation to follow strictly the rules of textbook writing competition, than a serious attempt to exploit the use of history of mathematics in teaching. This conclusion is supported by the existence of many serious errors, obscurities or omissions in the textbooks' historical texts, which make their use very problematic.

In the workshop we will present a number of examples that affirm this conclusion, we will indicate alternative ways to utilize all this historical material, but we will also propose different ways for exploiting the history of mathematics as a teaching tool.

Theme 3

Trade world's contribution to the spreading of mathematics education in Ioannina throughout the Ottoman period

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Thanks to special Ottoman decrees in the 15th century, Janina became a considerable trade and handicraft center in the Balkans in the middle of 18th century.

The Greeks of Diaspora trying to transfer the intellectual atmosphere of the West established some important schools (Gionma, Epiphanius, Maroutsia, Kaplanaia) and gave the opportunity to their students to receive a solid mathematical education.

The names of Balanos Vassilopoulos and Methodios Anthrakitis distinguished as representative of the Greek Enlightenment, constituted an important factor for the development of the mathematical education.

Theme 1

Investigation of the relationship between working memory, anxiety and performance during the

completion of an arithmetic mnemonic task

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It's well known that high anxiety levels affects performance during the completion of cognitive tasks, especially the difficult ones that require working memory resources. According to many researchers, anxious individuals occupy their memory with thoughts relevant and irrelevant to the task to be completed. These thoughts interfere with memory processes, reducing the available cognitive resources for elaboration and completion of the task to be completed. Having this theoretical framework in mind we explored the relationship between anxiety, cognitive interference and working memory functioning during the completion of an arithmetic memory task. For that purpose we used the following instruments: α) *Memory Subscale of the Wisc Intelligence Test*. In that subscale the child has to repeat a string of numbers. We read one number every one sec. The strings of numbers becomes longer and of increased difficulty, b) *State – Trait Anxiety Scale (STAI)*. This scale is consisted of 40 propositions and two scores are extracted from them: a) a score for *anxiety related to the environmental situations* (1-20 questions), b) a score for *anxiety related to dispositional factors* (21-40 questions), c) *Cognitive Interference Questionnaire (CIQ)*, consisted of 22 propositions. Three scores are extracted from the questionnaire: a) a score for thoughts relevant to the task to be completed (1-10 questions), b) a score for irrelevant to the task to be completed thoughts (11-21 questions), c) and a general question referring to “mind wondering”. This questionnaire refers to the kind of thoughts that go round the person's mind e.g. while they are completing a memory task.

We individually tested 204 children aged 10 – 12 years old. The analysis of data showed class differences concerning the above variables. Also, the linear regression analysis showed that task relevant and irrelevant thoughts participate in the formation of state and trait anxiety. The data are explained and compared to other findings from studies in Greece and abroad. The authors consider, as a result of anxiety's functioning, a possible mechanism underlying the malfunctioning of the Central Executive Subsystem of the Working Memory. At last, we consider that individual differences at anxiety levels deserve a better empirical investigation, especially during the functioning of Working Memory.

Theme 2

From real life to probability laws: Integrating problem solving in teaching statistics and probabilities

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In this paper, we report a modality to design a series of textbooks in statistics and probability. This series contains textbooks for high school and an introductory course for undergraduate students. We started our curriculum research from the following questions: How could teachers be influenced to change the teaching paradigm from one, which is based on formal definitions and properties to another that is more real life oriented? What specific tasks could raise the quality of students' learning of statistic and probabilities? An equally important issue that we have challenged was to succeed coming back to a formal meaningful presentation of the content when starting from real life problems. We hypothesize that confronting students with problems that he/she encounters in everyday life, we generate in students self-directed and active learning. This should also raise the interest in studying domains of mathematics such as statistics and probabilities, which are naturally more connected to the real world than other mathematical domains. Mass-media presents a variety of statistical information. Specific understanding

and procedures are needed to de-code this information in order to use it appropriately. By using statistical data, more general results can be expressed as probabilistic laws. As an illustration, for defining conditional probabilities, we started from a survey, which we used further in analyzing probabilistic properties and solving specific problems.

Theme 3

Effets d'un changement de programme sur la prise en compte de nouveaux objets dans l'enseignement des fonctions

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Depuis le début de la contre-réforme des mathématiques modernes, l'enseignement de la notion de fonction au début du lycée en France a subi de profondes mutations. Une des tendances les plus importantes concerne le renforcement progressif de l'utilisation des divers modes de représentation des fonctions. Ainsi parallèlement à une diminution de la suprématie du registre algébrique, le registre graphique a acquis de nouveaux droits et il y a également une injonction forte à utiliser dans des conditions nouvelles les objets tableaux de valeurs et de variations. Dans cet article, nous tentons de caractériser comment sont pris en compte, par les différents acteurs de l'institution, ces récentes modifications, en particulier concernant l'usage qui est fait des courbes, des tableaux de valeurs et des tableaux de variations dans le chapitre des généralités sur les fonctions en classe de seconde.

Theme 5

Problem posing through correspondence seminars

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A 'correspondence seminar' is one of the mathematical competitions organised in the Czech Republic for pupils of all types and levels of schools. A group of older students (secondary or university ones) prepares problems enclosed within a (real-life) story and sends them to younger pupils (by mail or e-mail). Later, they collect their solutions, check them and prepare another round of the competition. All is done via (e)-mail; that is why it is called a correspondence seminar. The article is to point out a possibility of using a correspondence form of work not only in the work with talented pupils but also in the everyday mathematics classroom to increase pupils' motivation towards mathematics and improve their problem solving skills. The competition is characterised and an illustration is given of one round of a correspondence seminar. The importance of the connection between problem posing and problem solving in schools is emphasised and the preparation of the correspondence seminar is presented as one of the ways of developing problem posing skills in university students, future mathematics teachers.